

Marking Streets to Improve Parking Density

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November 15, 2015

Abstract

Street parking spots for automobiles are a scarce commodity in most urban environments. The heterogeneity of car sizes makes it inefficient to rigidly define fixed-sized spots. Instead, unmarked streets in cities like New York leave placement decisions to individual drivers, who have no direct incentive to maximize street utilization.

This paper explores the effectiveness of two different behavioral interventions designed to encourage better parking, namely (1) educational campaigns to encourage drivers to “kiss the bumper” and reduce the distance between themselves and their neighbors, or (2) painting appropriately-spaced markings on the street and urging drivers to “hit the line”. Through analysis and simulation, the paper establishes that the greatest densities are achieved when lines are painted to create spots roughly twice of the expected cars length and the minimum gap between two adjacent cars. Kiss-the-bumper campaigns prove more effective for equal degrees of compliance on the street segments above 130 feet in length, but hit-the-line dominates on shorter street segments.

1 Introduction

Alternate side of the street parking is a unique but important New York City institution. Most city streets are assigned two intervals per week (typically 1.5 hours each) during which a particular side of the street must be vacated to allow for street cleaning. Drivers double park on the other side of the street during this time window, waiting for the moment when the street cleaner passes or the period expires, at which point they must quickly move their cars to newly clean and once-again legal spots [21]. Alternate side of the street parking defines the rhythm of life for many city residents [18] by mandating regular actions in order to maintain a car in the city.

The effective number of New York City parking spots depend heavily on the dynamics of alternate side of the street parking, since all cars park simultaneously but generally depart at distinct times. Thus it is rare for two adjacent spots to open simultaneously after the street configuration is frozen at the end of the forbidden interval. This means that any extra space left between the hastily-parked cars cannot be reclaimed until all the vehicles move again during the next parking switch. New York streets do not contain any laws, painted lines or boundary markings to guide positioning or restrict the space between neighboring cars after this transition. Since it is usually easier for the driver to leave ample room between cars, much of the potential parking area is wasted.

The upshot is that a classical random process fairly accurately captures parking behavior in the city, and motivates the challenge of identifying behavioral perturbations which significantly improve it. In this paper, we study the efficacy of two possible perturbations: (1) educational campaigns that encourage drivers to “kiss the bumper” and reduce the distance between themselves and their neighbors, or (2) painting appropriately-spaced markings on the street and urging drivers to “hit the line”.

This paper demonstrates that both mechanisms lead to increasing density as greater fractions of the population employ them. Our main contributions are:

- *Optimizing Line Spacing* – Through analysis and simulation, it establish that the greatest densities are achieved when lines are painted to create spots roughly twice the length of average-sized cars.
- *Relative Effectiveness of Interventions* – Kiss-the-bumper campaigns prove more effective than hit-the-line for equal degrees of compliance on street segment lengths above 130 feet, but hit-the-line dominates on shorter segments,

We believe that the results have genuine implications for improving the parking density of New York streets.

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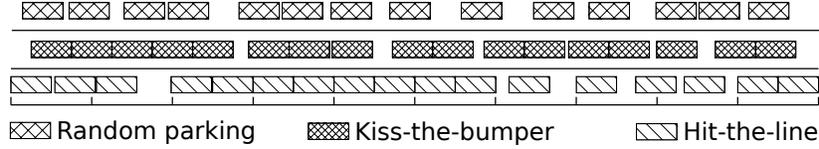


Figure 3.1: Representative street landscapes for random/Rényi (top), kiss-the-bumper (center), and hit-the-line (bottom) for a street of length 20 units with the optimal line spacing of 2 units, for compliance values of 0, 0.5, and 0.5 respectively.

2 Related Work

There is no substantial literature analyzing the mechanisms or dynamics of *urban* street parking. Municipal codes typically specify the required dimensions of on-street parking, but do not describe the rationale behind such a selection. Representative is the City of Bowling Green, Kentucky [6], which enforces an on-street parking layout that divide the street into even parking slots of size 6.7m to 7.9m, designed to leave at least 2.4m of maneuver space. Different cities have different regulations for on-street parking, although all must meet the American Disability Act (ADA) standards for accessibility [19]. Design guides to building multistory parking garage structures include [1, 5] Other studies seek to optimize the revenue from parking facilities [15].

Perhaps the most related paper in terms of research is [4], who study the performance of two different driver search strategies in minimizing (1) walking time, (2) driving time, and (3) combined time in reaching their destination in a typical parking lot layout. Like us, they are interested in analyzing the impact of drivers parking strategies. However, this paper concerns with maximizing the utilization of scarce street parking in an urban setting.

There has been considerable theoretical work on one particular model of street parking. The Rényi parking problem [14] concerns the following random process. Unit-length cars select unoccupied positions on the real line uniformly at random, starting from an initially empty interval of length l . The process continues until no vacant unit-length interval remains.

Each car parked in an open interval splits the interval into two smaller ones, each independent from the other. Therefore this random process can be analyzed by a recursive formula. Let $f(l)$ denote the expected number of cars parked on a street of length l . Then $f(l)$ is determined by the following delay differential equation:

$$f(l) = 1 + \frac{1}{l-1} \int_0^{l-1} (f(x) + f(l-x-1)) dx = 1 + \frac{2}{l-1} \int_0^{l-1} f(x) dx \quad (1)$$

with the base cases $f(l) = 0$ for $l \in [0, 1)$ [8].

The parking density is defined as $f(l)/l$, with the Rényi parking constant giving the limit of density as $l \rightarrow \infty$. Numerical calculations show that

$$\lim_{l \rightarrow \infty} f(l)/l \approx 0.7476$$

For small integral values of l , exact values of $f(l)$ are known, including $f(1) = 1$, $f(2) = 1/2$, $f(3) = 2/3$, and $f(4) = (11 - 4 \ln 2)/3$. The variance in this process has been determined [8, 12]. Recently, the expected number of gaps of certain size was analyzed for a discrete version of the parking problem [7].

The problem has been generalized into higher dimensions. Here the goal is to “park” unit n -cubes into a larger n -dimensional cube. Results in two dimensions include [2, 13]. Higher-dimensional versions are widely studied in statistical physics as the random sequential adsorption problem. It is investigated as the end configuration after molecules attaching itself to a surface. See [3] for a survey of the field. The limit of the parking density in n -dimensions is a difficult problem, and even the two-dimensional version remains open [9].

These investigations suggest a new class of bin packing or knapsack problems [10, 11] where the heuristic employed are not under central control, but instead the heuristic is selected at random to position each additional element.

3 Models of Parking Behavior

Painted street markings are the traditional way to enforce parking behavior. Each slot must accommodate the largest-sized vehicle on the road (e.g. the GMC Savana van, 224 inches (5.6896 meters) long), leaving considerable

wasted space when small vehicles (e.g. the Nissan Cube, 158 inches (4.0132 meters)) are parked. The longest cars on the road approach twice the length of the shortest, making maximally-generous spots too wasteful for the city to enforce.

Instead, one can consider lines as *optional* guide markings to help drivers make better decisions on where to place their car. These lines serve the same role as etching images of flies in men’s urinals: providing a target to hit that encourages better behavior. Schiphol Airport in Amsterdam reports a 80% reduction in spillage as a result of this design [20]. The increasing prevalence of cars capable of automated on-street parallel parking [17] provide an additional incentive, as the associated computer vision algorithms also take advantage of line markings.

The paper concerns three basic parking strategies (illustrated in (Figure 3.1)) where observing street markings is optional:

- *Random parking* – Consistent with the Rényi model, the driver selects a location uniformly at random over all unoccupied locations on the given street. We believe this is a fairly accurate model of what happens in practice during the phase transition from double parking at the instant the other side of the street becomes legal again.
- *Kiss the bumper* – Here a driver selects a location uniformly at random and moves up to leave minimum space between the neighboring car. Uniformly observed, this strategy provides optimal street utilization, but the parallel maneuvering during the shift makes it impossible to enforce such behavior.
- *Hit the line* – Here a driver selects an unoccupied street marking (line) uniformly at random and lines up immediately behind it. In the event that all lines are occupied, the kiss the bumper strategy is employed.

Consider the situation when the population employs a pair of these basic strategies, typically the random/Rényi strategy and a more sophisticated algorithm. Let α denote that fraction of the population employing the more sophisticated strategy, i.e. the “good” drivers. The expected density of cars parked on the street depends upon α . For $\alpha = 0$, all drivers are random/Rényi drivers, so the achievable density will be 0.7476. For $\alpha = 1$, all drivers behave in a socially-minded way, and the resulting density approaches the optimal value of 1.0.

The Rényi recurrence relation can be readily generalized to deal with a mix of strategies for unit length cars. Let $f(l)$ denote the expected number of cars that will be parked on a street of length l where the α -fraction of good drivers employ the kiss-the-bumper strategy. Then

$$f(l) = 1 + \alpha f(l-1) + (1-\alpha) \frac{2}{l-1} \int_0^{l-1} f(x) dx \quad (2)$$

Alternately, let $f_k(l, t)$ denote the expected number of cars that will be parked on a street of length l where an α -fraction of good drivers employ hit-the-line. The street is painted with lines k units apart, such the initial line is t units away from the origin. Then

$$f_k(l, t) = 1 + \alpha f(l-1) + (1-\alpha) \frac{1}{l-1} \int_0^{l-1} f_k(x, \min(x, t)) + g_k(l, x, t) dx \quad (3)$$

$$g_k(l, x, t) = \begin{cases} f_k(l-x-1, t-x-1) & \text{if } x+1 \leq t \\ f_k(l-x-1, k - ((x+1-t) - k \lfloor \frac{x+1-t}{k} \rfloor)) & \text{if } t < x+1 \leq t + k \lfloor \frac{l-t}{k} \rfloor \\ f_k(l-x-1, l-x-1) & \text{if } t + k \lfloor \frac{l-t}{k} \rfloor < x+1 \end{cases} \quad (4)$$

$$f_k(l, t) = f(l) = 0 \text{ if } 0 \leq l < 1 \quad (5)$$

These equations are difficult to solve both numerically and analytically. Therefore discrete-event simulations was employed to study to performance of particular strategy mixes.

4 Line Spacing for Unit-Length Cars

Consider the problem of spacing painted lines so as to induce the greatest expected parking density as a function of α . The street markings occur regularly every k units over the length l street. For unit-length cars, and compliance fraction $\alpha = 1$, clearly $k = 1$ is optimal. However, this is not the case when there is lower compliance, and the $(1-\alpha)$ fraction of “bad” drivers employ the Rényi strategy.

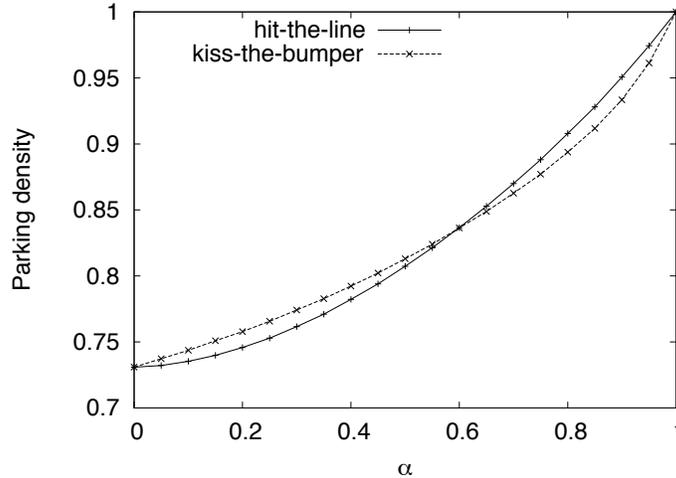


Figure 4.1: Parking density of kiss-the-bumper and hit-the-line as a function of α , for the optimal line spacing of $k = 2$ and $l = 20$.

In particular, through simulation, one can compare the expected density as a function of k and α . One can also compare it to the benchmark kiss the bumper strategy for the same value of α . For each strategy, the simulations were run on $l = 20$ with 100,000 repetitions on 200 evenly spaced data points of α from 0 to 1. Each data point for a specific α represent the average of the parking density over all repetitions.

The primary result is shown in Figure 4.1. First observe that the parking density of both the kiss-the-bumper and hit-the-line strategies range from the Rényi parking constant to an optimal packing as the compliance constant α ranges from zero to one. The density increases in a non-linear fashion for both strategies, with densities of 0.82 achieved by $\alpha = 0.5$. This is only about 25% of the total density improvement with $\alpha = 1$, so relatively high compliance rates must be achieved to substantially increased density.

The behavior of the two strategies are similar but not identical. Kiss-the-bumper outperforms hit-the-line until a compliance threshold of approximately $\alpha = 0.592$, beyond which hit-the-line is better. Here, the optimal line spacing were used. Also, it's assumed that “good” drivers kiss-the-bumper when no line is available to hit.

Figure 4.2(a) explores parking density as a function of k , the number of car widths between painted lines. Surprisingly, the best performance is achieved when painting lines between every *other* spot, (i.e. $k = 2$), as opposed to the standard practice of delimiting the precise boundary of every car. The reason becomes clear in hindsight. Two “good” drivers parked on consecutive lines for $k = 2$ will create a unit-length pocket between them, achieving optimal density even if eventually taken up by a normally uncooperative driver.

This mechanism also explains why the relative advantage of painting lines accrues primarily for large values of α : enough lines must be hit to force such tight pockets. Unit-length pockets are also occasionally created for other painting patterns (i.e. $k \neq 2$), but the probability of creating them proves too small to compete with the benchmark strategy.

Larger gaps between painted lines necessarily implies fewer lines available to hit. Once these are exhausted, the “good” drivers must shift to an alternate strategy. Figure 4.2 contrasts the densities achieved with kiss-the-bumper as a secondary strategy (left) vs. random/Rényi (right). The interactions between Rényi and painted lines can be disastrous: worse than no lines at all for $k = 3$. There the good drivers tend create pockets of two spaces, one of which becomes permanently lost to a Rényi driver.

In general, much of the increase in parking density from line painting comes from kiss-the-bumper as the secondary strategy. The painted lines primarily help by creating unit-length spaces. Let $f(x)$ denote the number of cars a open space of length x eventually contains. After a bumper-kiss event, $f(x) = 1 + f(x - 1)$. But a good driver hitting the line at position y , yields $f(x) = 1 + f(y) + f(x - y - 1)$, which is similar to the effect of a Rényi driver.

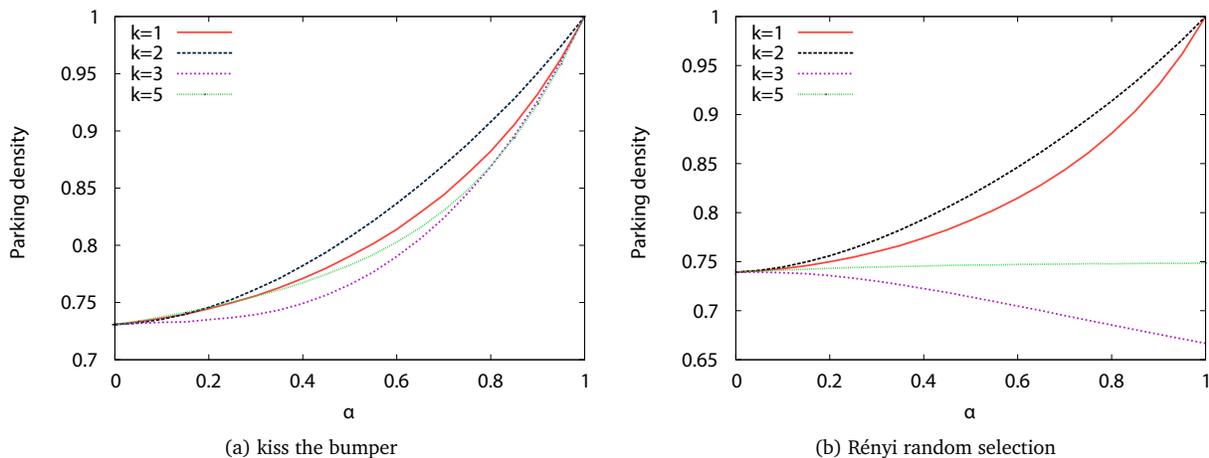


Figure 4.2: Effect of secondary strategies as a function of line separation for line separations of 1, 2, 3, and 5 car lengths.

5 First-Available Space Strategies

The previous section posits drivers who exhibit no preferences when faced with the Nirvana of multiple open spots on the street. This utility function accurately models the behavior in the shift from double-parking to clean streets, since the double-parked cars are uniformly distributed along the street. It also seems relevant to the case where drivers have random destinations on the street after they leave their vehicle.

However, in many situations drivers are likely to pounce on the first available space they see as they enter the street. With the new model, normal drivers will pick a random position in the first open space. The driver's behavior is now limited to the first open pocket (i.e. the pocket closest to $x = 0$) of sufficient size to hold their unit-length vehicle. Again, assume that a $(1 - \alpha)$ fraction of drivers employ the Rényi strategy, while the more civic-minded α fraction pursue one of three beneficial approaches:

- *Kiss the Bumper* – Such drivers park directly behind the top car delimiting the first open pocket.
- *Hit the Line* – Such drivers park on the first line they see in the first open pocket, if one exists. Otherwise, they employ kiss-the-bumper.
- *Crave the Line* – These drivers go the extra length to park at the first line they see, even if it's not in the first open pocket. They kiss-the-bumper if they don't find any unoccupied lines. Intuitively, a sequence of k crave-the-line drivers will create a sequence of $k - 1$ open spaces of unit-length, which when occupied by arbitrary drivers provides optimal density over the interval of length $2k - 1$.

The results are shown in Figure 5.1(a). This figure, comparing hit-the-line to kiss-the-bumper, looks very similar to Figure 4.1. Thus driver preferences for first pocket instead of random pockets have no significant implication on the design of line spacing or behavioral interventions.

More surprising is the excellent performance of crave the line, shown in Figure 5.1(b). The increase in parking density grows rapidly once one achieves a compliance rate of roughly $\alpha = 0.25$. Although one should be skeptical that large numbers of drivers can be convinced to pass an open spot to hit the next, the results do speak of the value of emphasizing proper line markings.

6 Variable-Sized Cars

The results reported thus far all assume unit-length vehicles, thus ignoring the natural diversity of car sizes from compacts to SUVs. But this non-uniform size distribution is an important factor governing urban street parking dynamics. Certainly smaller cars have an easier time finding a parking in the city than big ones, which goes a long way towards explaining the popularity of half-size Smart cars in urban environments.

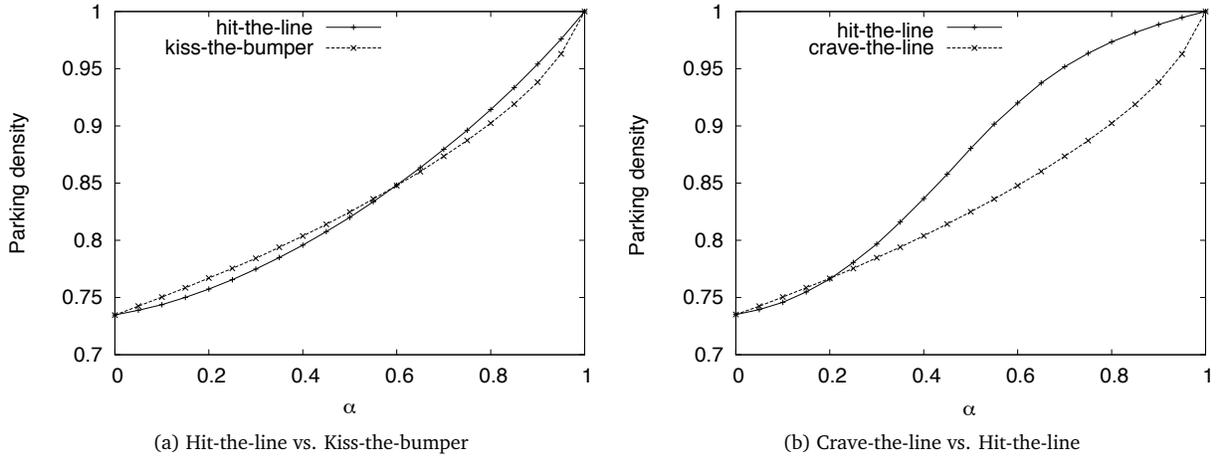


Figure 5.1: Parking densities for hit-the-line, crave-the-line, and kiss-the bumper restricted to the first-available pocket.

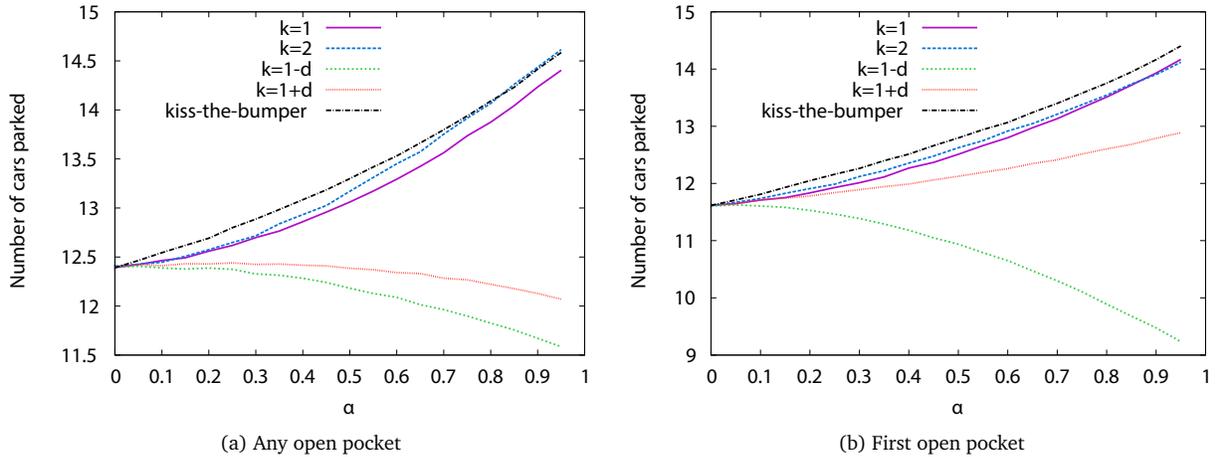


Figure 6.1: Parking densities for car lengths drawn uniformly from $[0.75, 1.25]$.

Variable car lengths require us to consider whether the optimization criteria is to maximize the number of cars resting on the street, or the fraction of the total street length being utilized by parked cars. The former could best be done by only allowing half-sized cars to park, while the later would reserve a spot for the largest car which could fit in it. This paper seeks to maximize the number of parked vehicles without explicitly favoring small cars over large ones. Thus cars are served at random as per their length-frequency distribution, but the process continues until no more empty spots for minimum-length cars remain.

In this section, the two different models of non-uniform car lengths are compared. First, the model where cars are uniformly distributed over a size range. Then, the model with actual car sales data and representative Manhattan streets to strive for more accurate modeling.

6.1 Uniform Distribution of Lengths

To capture the variance of car lengths, the car lengths are generated by a uniform distribution in the range $[1 - d, 1 + d]$, for some value of d . The previously studied case of unit-length cars corresponds to $d = 0$.

The four gap sizes for the line markings for each of the three pocket-selection models previously described (random pocket, first pocket, and crave-the-line) were evaluated. The gap sizes are $1 - d$, $1 + d$, 1 or 2 .

The results for $d = 0.25$ are shown in Figure 6.1, with the results for $d = 0.125$ and $d = 0.5$ quite similar. As expected, the $k = 2$ gap length generally outperforms all other line markings, although there are small windows

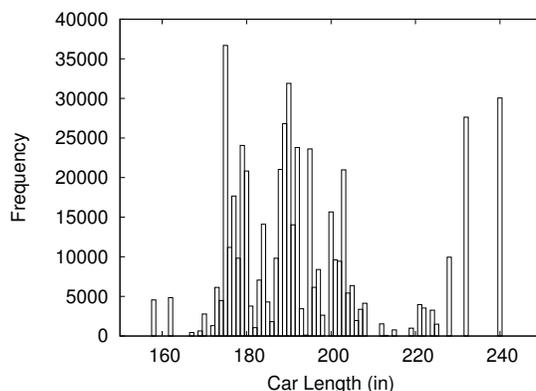


Figure 6.2: North American car sales in January 2010 as a function of body length.

where other spacings dominate for high α in hitting the line. Making the gap larger or smaller than an integral size of the car did not perform well. Indeed the tight gap spacing of $1 - d$ results in strictly fewer parked cars as compliance increases.

6.2 Real World Distributions

The optimal line spacing depends on the distribution of the car lengths driven by local residents. To better capture the dynamics of real world parking, new car sales figures was used as a proxy for the length distribution in use on today’s roads. In particular, we compiled the January 2010 sales for each current model from GM, Ford, Honda, Toyota, Nissan, Chrysler, and Kia. These seven companies together control 85% of the automobile market share in North America [16]. Figure 6.2 presents the relative frequencies of cars by body length. The average car length is 194.4in. It is strikingly irregular, and not particularly well approximated by the uniform distribution employed in the previous section.

Two Manhattan streets of typical size are the Broadway-to-Amsterdam and Amsterdam-to-Columbus blocks of West 92nd Street, which measure in at 340 feet and 831 feet, respectively. Unfortunately, fire-hydrants, driveways and no-parking zones reduce the parking spaces, and partition the streets to curbs of a much shorter length. For example, the north side of Amsterdam-to-Columbus block contains three fire-hydrants, a single driveway, and a no-parking zone. Redefining “street” to be a maximal contiguous block of curb that allows parking avoids this complexity. Our simulations were performed on street segments of length at most 200 feet. We enforce that adjacent cars be spaced at least one foot (0.305 meters) apart from each other. Our good driver strategy is hit-the-line, with secondary strategy of kiss-the-bumper.

Recall that mathematical statements concerning Renyi parking model are asymptotic, meaning they hold only when the length of the street approaches infinity. Unexpected behaviors can occur on short street segments.

Indeed, small tweaks can make a big difference in utilization of short segments. We propose to paint the line at the beginning of the street, but after a gap the length of a small car. This implement a *smart car zone* of 170 in at the beginning of the curb. In the best case, with proper street marking, we can expect a hit-the-line driver to hit the first line, and leaving room for one small car at the beginning of the curb.

We ran the simulation is on streets of various lengths. Each data point in Figures 6.3 and 6.4 are generated through simulations of 100,000 instances where ℓ, k, α and strategy are fixed. Each instance generates a street of length ℓ with marking distance k . Cars are sampled from the size distribution, and contains a good driver with probability α . Each car enters the street at the start and follows the prescribed parking strategy. The process terminates as soon as the shortest car cannot find a parking space.

Figure 6.3 presents the expected number of parked cars on various street lengths as a function of the gap between painted lines for four values of α .

all the curves on streets above 50 feet are bimodal, with two peaks and a sharp dip in between. This reflect whether segment length has capacity for one or two cars. At 50 feet, the street does flattens out because it does not have enough space to make larger gaps meaningful.

Regardless of the compliance level or the length of the street, the optimal gap length peaks between 375in and 425in, with little variance inside the range. Thus the optimal marking length proves robust to compliance level and street segment length. Too short gaps significantly degrade performance, to a point well below that of

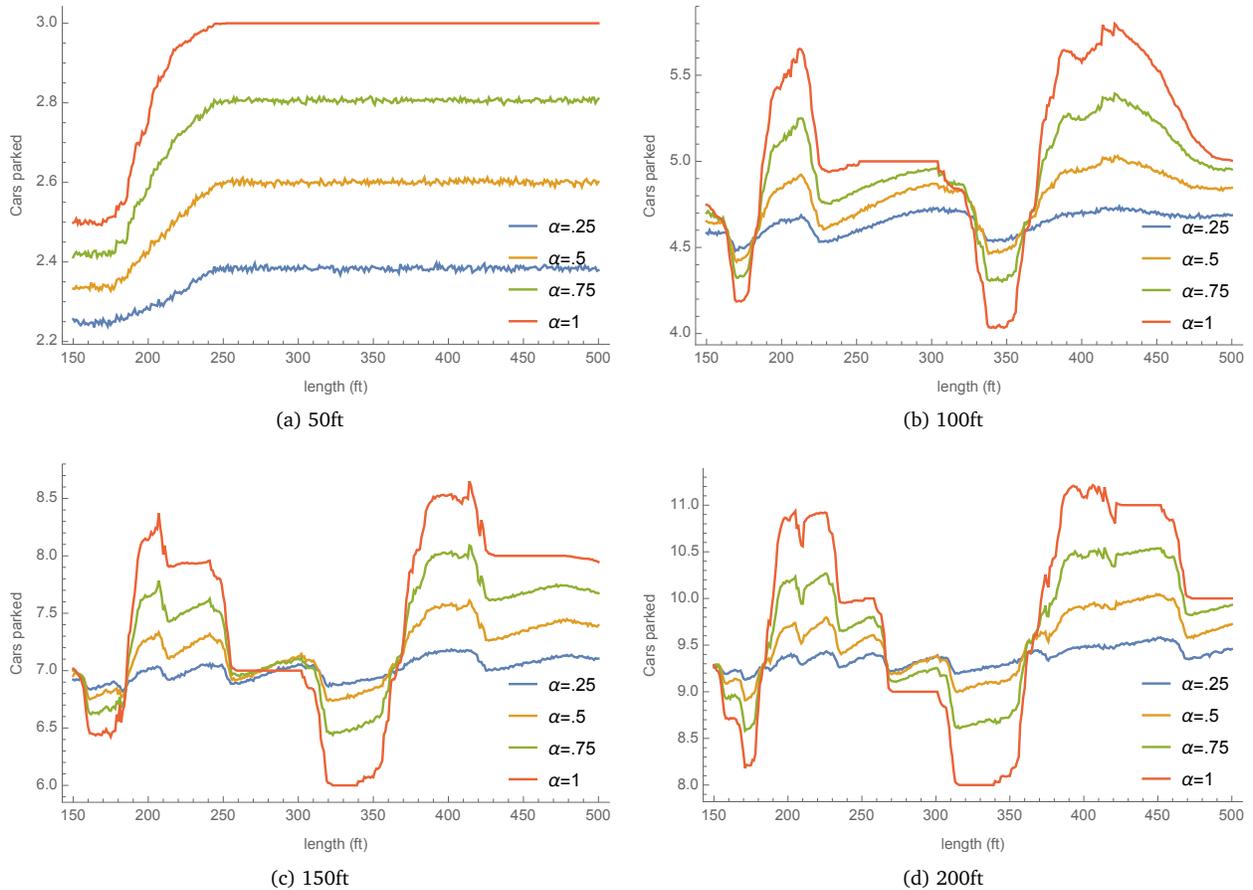


Figure 6.3: Simulation result for real world data: the number of parked cars using hit-the-line strategy as a function of the distance between painted lines for street lengths 50 ft, 100 ft, 150 ft and 200 ft for α values of .25, .5, .75 and 1.

unmarked streets. The magnitude of this oscillatory behavior increases strictly with α . As previously observed, optimal density is achieved when the gap is roughly twice the sum of average car length (413 inches).

Figure 6.4 provides an alternate view of this data, showing the average number of cars parked on different street lengths as a function of α , for line-spacings 422 inches apart with a 170 inch smart car zone. The general trend is the number of cars increases continually with greater compliance.

For short street segments with properly spaced lines, hit-the-line fares better than kiss-the-bumper strategy at the same level of compliance. However, for longer street segments, kiss-the-bumper becomes the dominating strategy. Our experiments indicate that the street segment length where the two strategies switch is approximately 130 feet.

It seems counter-intuitive that hit-the-line can kiss-the-bumper can out-perform hit-the-line when there is perfect compliance, since kiss-the-bumper should achieve almost complete utilization of the curb.

But kiss-the-bumper can favor neighboring large cars, which prove disastrous on short street segments. This effect dies out as the curb length increases to street segments of length 150 ft and 200 ft.

The difference between two strategies are small even on long streets. Indeed, given an additional 0.1 probability of hit-the-line compliance on a 200 ft street would dominates kiss-the-bumper.

We have released the source code for our simulation on GitHub [22] for those who would like to perform additional experiments.

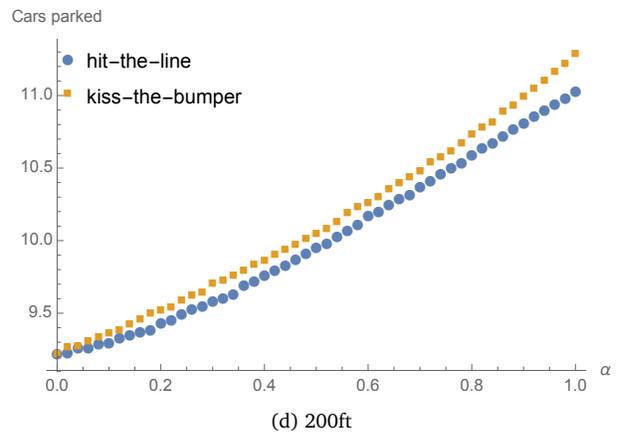
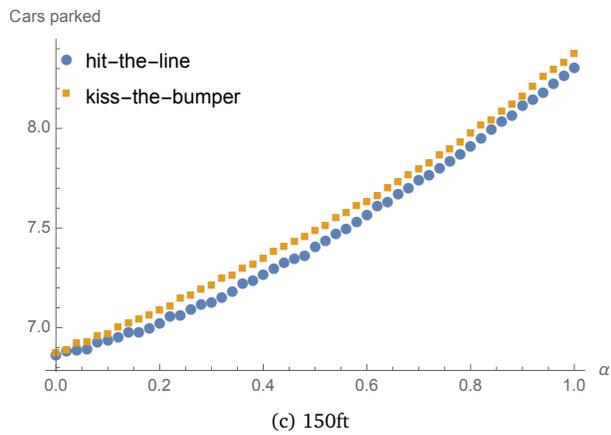
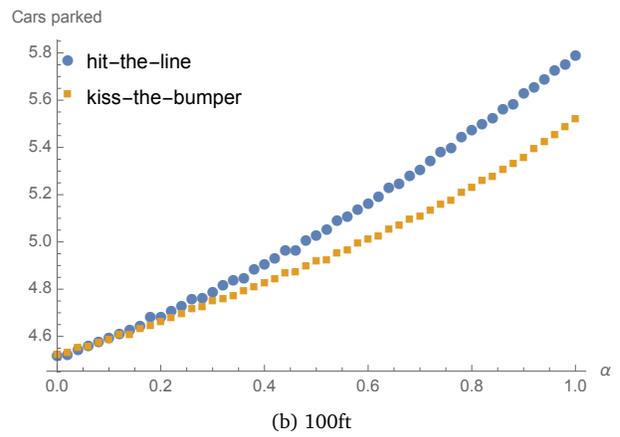
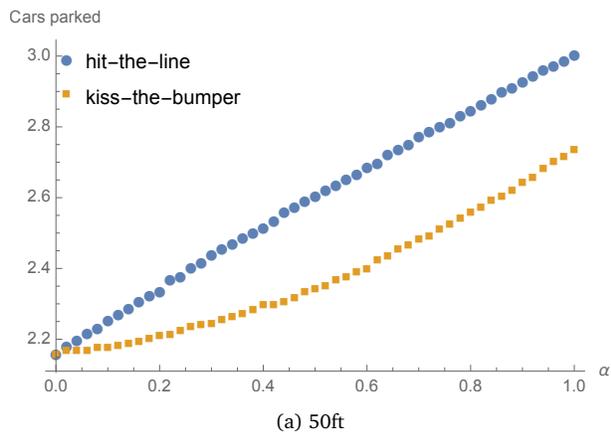


Figure 6.4: Simulation result for real world data: the number of parked cars as a function of the compliance rate α for street lengths 50 ft, 100 ft, 150 ft and 200 ft.

7 Conclusions

The paper studies the impact of two behavioral interventions (encouraging kiss-the-bumper or hit-the-line parking driving) to improve space utilization in city streets. Generally kiss-the-bumper provides better utilization for a given level of compliance for curbs of 130 ft or more, but hit-the-line performs better for shorter curbs.

We presume that the absence of visual cues makes it difficult to alter current parking habits. However, if painting guide lines can increase compliance (α) by only 0.1, then painting guide lines two car-widths apart is the right strategy to optimize resource utilization.

With the proper street markings, we anticipate squeezing in one additional car per three street segments of length 50 feet, when $\alpha = 0.4$. For longer segments, we expect to squeeze in one extra car per two streets of length 200 ft if α improves by 0.1 in response to visual cues.

Acknowledgments

The authors wish to thank the Algorithms Reading Group at Stony Brook for helpful discussions. The first author was supported in part by National Science Foundation Research Experiences for Undergraduates supplement IIS-1128741. The second author was supported in part by National Science Foundation grants IIS-1017181 and DBI-1060572. We thank the referees for suggestions which improved the results and presentation of this paper.

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